

MATHEMATICS COLLOQUIUM

Vector Products and Division Algebras

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Abstract: The cross product of vectors in Euclidean 3-space can be introduced in undergraduate linear algebra courses. One might wonder why contrary to the dot product, which is available for any finitedimensional real vector space, the cross product is only defined for 3-space. In my talk I will discuss the existence of cross products of vectors in Euclidean n-space for arbitrary n. It turns out that this is only the case if n is 0, 1, 3, or 7.

This result is closely related to a theorem of Hurwitz on the existence of composition algebras over the real numbers (1898) and to a theorem of Zorn on the structure of finite-dimensional alternative division algebras over the real numbers (1933). Both kinds of algebras only exist in dimensions 1, 2, 4, or 8 (and are realized by the real numbers, the complex numbers, Hamilton's quaternions, and Cayley's octonions, respectively). All proofs are very elementary and can be done in a purely algebraic manner. The goal of my talk is to show the intimate relation between these algebraic structures and their classification. If time permits, I will also indicate the relevance of topology to the classification of more general (i.e., non-alternative) real division algebras.

2010 Mathematics Subject Classification: 17A35, 17A45, 17A75, 17A60

Date: **Thursday, October 27, 2016** Time: 12:15pm-1:15pm Place: MAGC 1.302

Refreshments will be served at 12:00pm

For further information, or for special accommodations contact Dr. Zhijun Qiao via email at zhijun.qiao@utrgv.edu or at 956-665-3406.